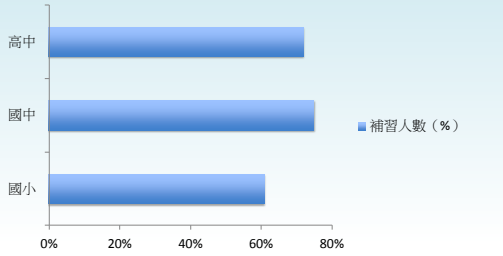
腦中的數學 - 從fMRI研究談數學認知發展

張葶葶
國立政治大學心理系
國立政治大學心腦學中心

Overview

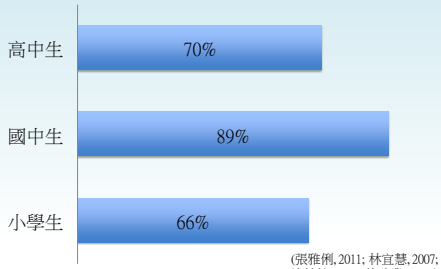
- How is mental arithmetic stored and processed in the brain?
- How does the neural network of mental arithmetic processing develop with learning and experience?
- How does atypical developing arithmetic skills represented in the brain?

台灣地區學生補習人數概況



整理自〈九十四年臺閩地區兒童及少年生活狀況調查報告〉

中小學生補習數學者佔補習人口比例



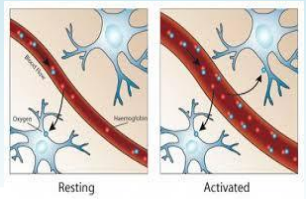
(張雅俐, 2011; 林宜慧, 2007; 許綺婷, 2001; 徐政業, 2008)

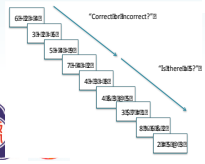
What is mathematical skill?

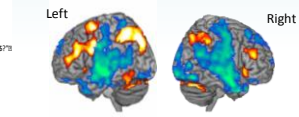
- Basic Number Processing
  - Number Representation
    - Magnitude Judgments
    - Symbols vs. Numbers
- Mathematical Computation
  - Arithmetic
  - Calculation

$3 + 6 = 9$
$17 - 9 = 8$

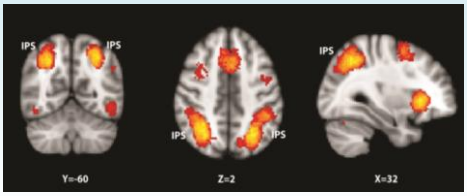
functional Magnetic Resonance Imaging





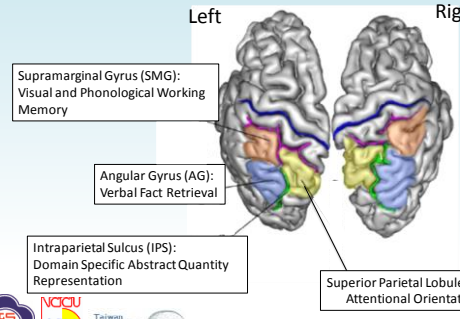


Canonical Brain Areas Involved in Arithmetic Problem Solving



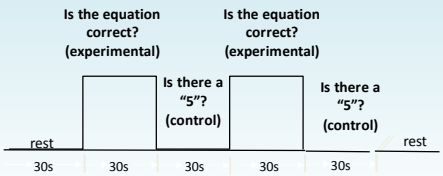
Maps are based on meta-analysis of 44 studies of arithmetic in neurosynth (Yarkoni et al. 2011).

Posterior Parietal Cortex (PPC)



Menon (2010), Dehaene et al., (2003)

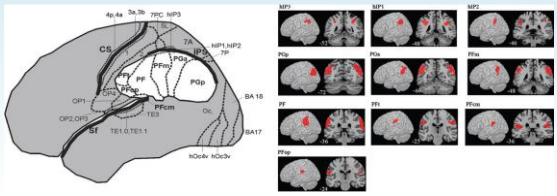
Mental arithmetic task



Arabic	3 + 2 – 1 = 5	6 @ 2 # 1 * 2
Roman	III + II – I = V	VI # II @ I & II

Wu, Chang et al. (2009), *Cerebral Cortex*

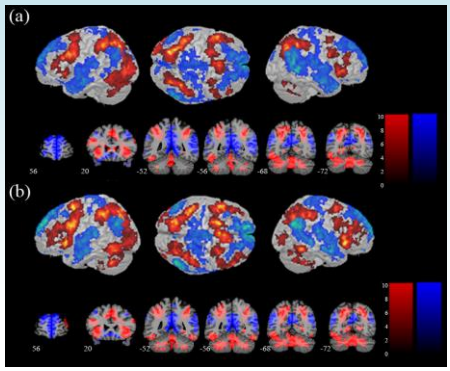
Cytoarchitectonic Probablistic Maps



Wu Chang et al. (2009), *Cerebral Cortex*

Arabic-control

Roman-control

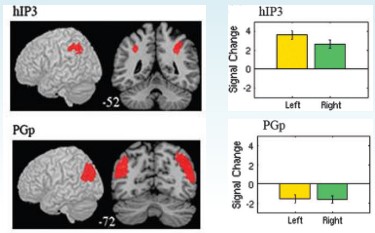


Wu, Chang et al. (2009), *Cerebral Cortex*

Distinct PPC profile

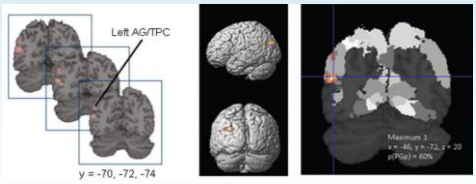
Arabic Numeral : "Is this correct?" 3 + 4 – 2 = 5

Roman Numeral : "Is this correct?" III + IV – II = V



Wu, Chang et al. (2009), *Cerebral Cortex*

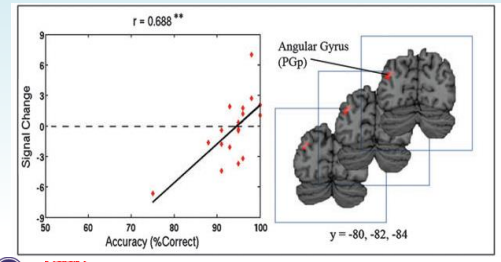
Activation Difference Between Arabic and Roman Numerals



Wu, Chang et al. (2009), *Cerebral Cortex*

Are the neural correlates of mental arithmetic modulated by mathematical competence?

AG activation Correlates with Accuracy



Wu, Chang et al. (2009), *Cerebral Cortex*

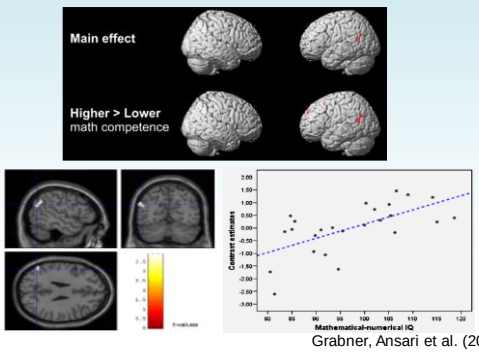
Another example

- Screened a large sample of adults (138)
- Selected *individuals who did not differ in IQ but varied in their mathematical competence*
- fMRI study
  - Multiplication verification **4 x 6 = 24**
  - Control Task **3 = 3 = 3**

Which brain regions activated during multiplication correlated with mathematical competence?

Grabner, Ansari et al. (2007)

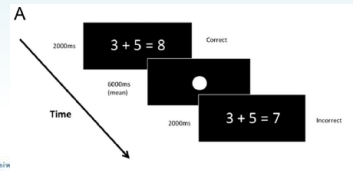
Relationship between AG activation and individual difference in math skill



Grabner, Ansari et al. (2007)

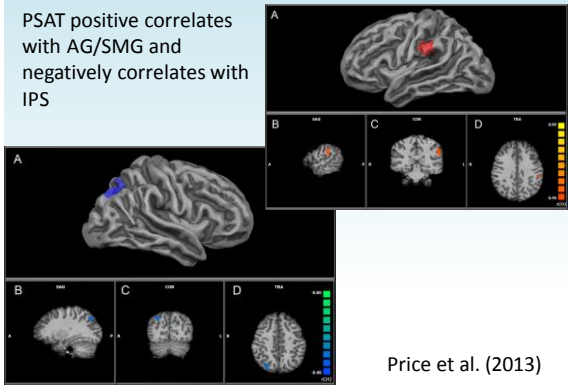
Brain activation predicts high school math

- Participants
  - 33 high school students (mean age :17 yrs)
- Math skill assessment
  - PSAT



Price et al. (2013)

PSAT positive correlates with AG/SMG and negatively correlates with IPS



Price et al. (2013)

Are the neural correlates of mental arithmetic modulated by strategy choice?

5 + 3 = ?

14 + 25 = ?

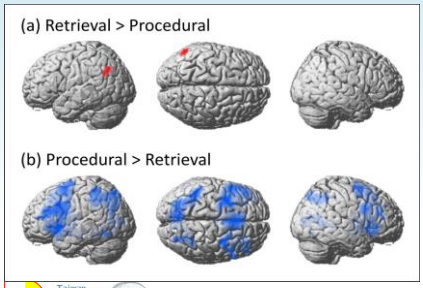
45 + 78 = ?

2874 + 3527 = ?

Arithmetic strategy

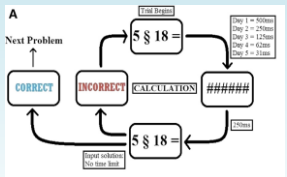
- Retrieval
  - Directly recollect answer in one step
- Procedural calculation
  - Calculate answer using explicit algorithm

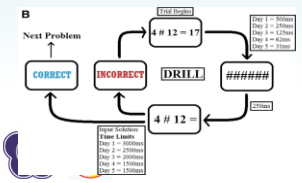
Strategy Variability
Evidence from Brain Imaging



Grabner, Ansari et al. (2009)

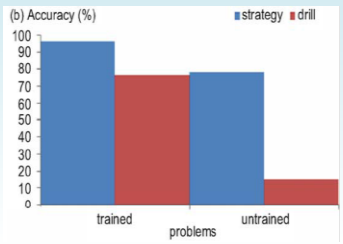
Learning by algorithm or learning by drill?





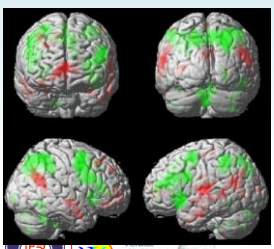
Procedure:
1. [(right number – left number) + 1] + right number
2. [(right number + left number) – 10] + right number

Delazer et al. (2005)

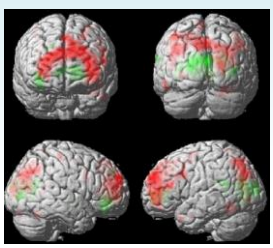


Delazer et al. (2005)

Training effect:
trained vs. untrained
untrained vs. trained

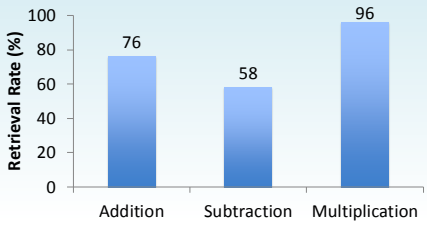


Strategy effect:
drill vs. algorithm
algorithm vs. drill



Does the brain activate differently across basic arithmetic operations?

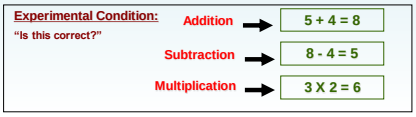
Problem solving strategies varies across arithmetic problems



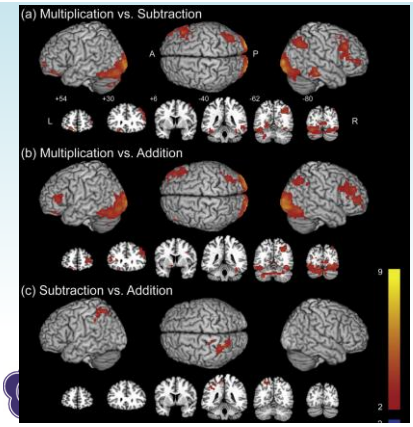
Campbell & Xue, 2001

Functional Dissociation Between Basic Arithmetic Operations

- **Participants**
  - 20 healthy adults (age 18-30)
- **Tasks**



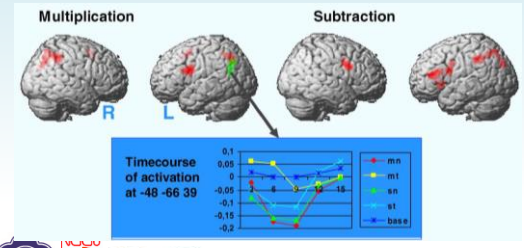
Rosenberg-Lee, Chang et al. (2011)



Rosenberg-Lee, Chang et al. (2011)

How specific do we learn?

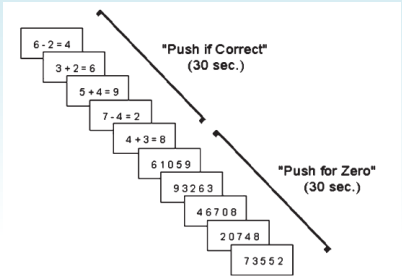
trained vs. untrained
untrained vs. trained



Ischebeck et al. (2006)

How does the neural network of mathematical information processing develop with learning and experience?

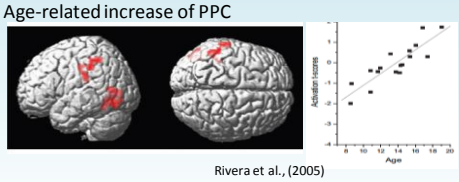
Development of mental arithmetic



Rivera et al. (2005)

Developmental Change of Mental Arithmetics

Age-related increase of PPC



Rivera et al., (2005)

Does all PPC subdivisions follow a heterogeneous or homogeneous linear developmental trajectory?

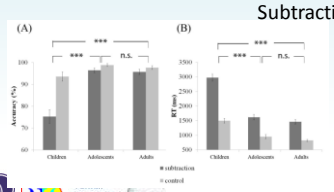
Is there nonlinear developmental change in the PPC?

Development of mental arithmetic across adolescence

- Participants
  - 25 children (age 7-10)
  - 19 adolescents (age 13-17)
  - 26 adults (age 19-22)

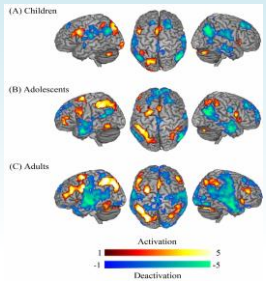
13 – 5 = 9
12 – 3 = 7
9 – 5 = 4
Subtraction

5 @ 1 $ 4
4 % 1 # 3
6 & 1 @ 7
Control



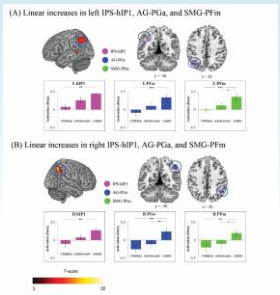
Chang et al. (under revision)

Activations and Deactivations in Children, Adolescents, and Adults



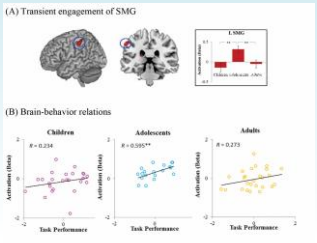
Chang, et al. (under revision)

PPC Regions Showing Linear Developmental Changes



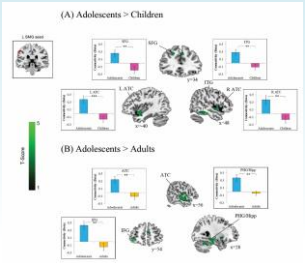
Chang, Metcalfe, Padmanabhan, Chen & Menon (under revision)

PPC Regions Showing Transient Engagement in Adolescents



Chang, Metcalfe, Padmanabhan, Chen & Menon (under revision)

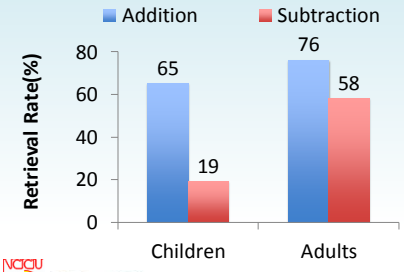
Nonlinear development of SMG connectivity



Chang, Metcalfe, Padmanabhan, Chen & Menon (under revision)

How about the development of different basic arithmetic operation?

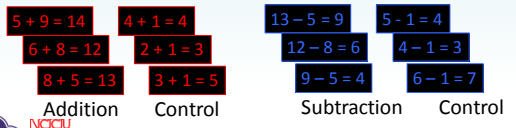
Arithmetic problem solving strategies converge across addition and subtraction



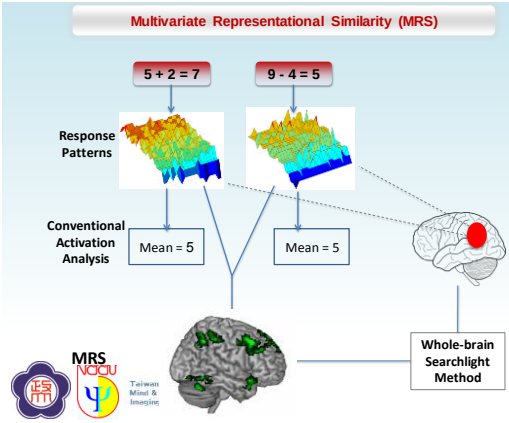
Campbell & Xue, 2001;Barrouillet, Mignon, & Thevenot, 2008

Experimental Design

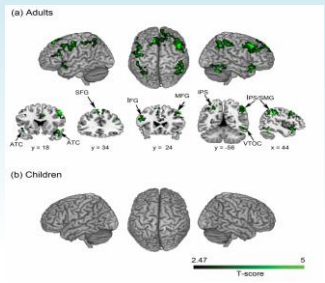
- Cross-sectional fMRI
  - 28 Children (7-9 yrs)
  - 28 Adults (18-22 yrs)
- Block design



Chang et al. (under revision)

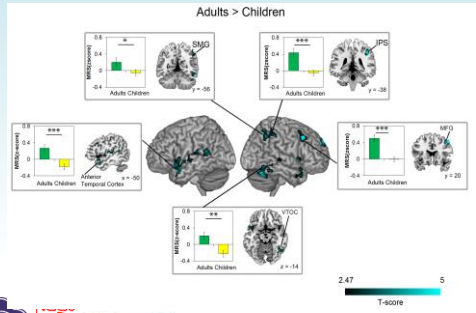


MRS Maps for Arithmetic Problems of Children and Adults



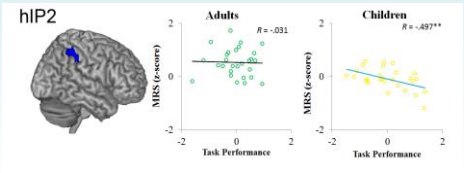
Chang, Rosenberg-Lee, Metcalfe, Chen, & Menon (under revision)

MRS - whole brain



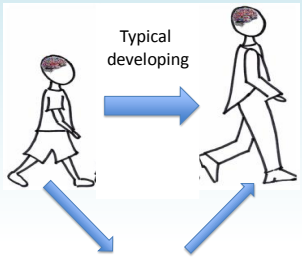
Chang et al. (under revision)

MRS in hIP2 correlates with task performance in children but not in adults



Chang et al. (under revision)

What about atypical developing?

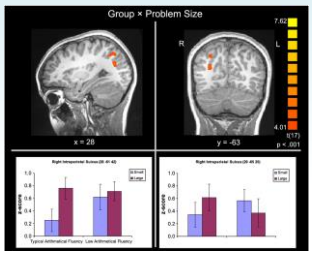


Developmental Dyscalculia (DD)

- DD is a specific learning disability affecting the acquisition of school-level mathematical abilities in the context of otherwise normal academic achievement, with prevalence rate of 3-6% (Price et al., 2007).
- DD children show persistent deficits in mathematical skill.
  - longitudinal study of 140 11-yr old children with DD (Shalev et al., 2005)
    - After 3 years, 95% of the group still meet DD criteria
    - After 6 years,
      - 51% could not solvle 7 X 8 (vs. 17% of controls)
      - 71% could not solve 37 X 24 (vs. 27%)
      - 49% could not solve 45 X 3 (vs. 15%)
      - 63% could not solve 5/9 + 2/9 (vs. 17%)

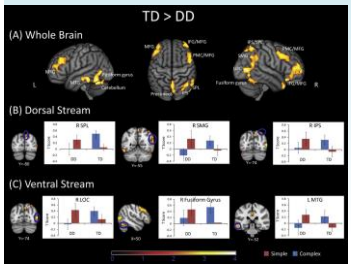
Children with low math skill

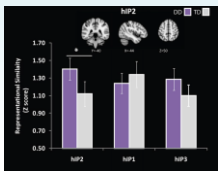
- fMRI study of complex and simple addition and subtraction problem
- 10-12 year old children



DeSmedt et al. (2011)

DD failed to show brain regions modulated by task complexity





Ashkenazi et al. (2012)

Neural Representation in Typical Developing (TD) and Developmental Dyscalculia (DD) Children

- Participants
  - Full scale IQ > 80
  - WIAT-II, NumOps
    - 21 TD, percentile > 25th
    - 16 DD, percentile <=25th
- Hypothesis
  - DD show weak distinct representation between problem types.

Task

5 + 9 = 14
6 + 8 = 12
8 + 5 = 13
4 + 1 = 4
2 + 1 = 3
3 + 1 = 5

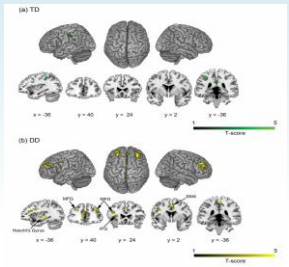
Addition Control

13 – 5 = 9
12 – 8 = 6
9 – 5 = 4
5 - 1 = 4
4 – 1 = 3
6 – 1 = 7

Subtraction Control

Chang, Iuculano, & Menon (in preparation)

MRS Maps for Arithmetic Problems of TD and DD children



Chang, Iuculano, & Menon (in preparation)

Summary

- PPC is consistently implicated in mental arithmetics.
  - PPC has distinct function in mathematical cognition.
  - PPC is modulated by mathematical competence and strategy use.
- Development profile of PPC
  - developmental shift from PFC to PPC in mathematical cognition
  - Heterogeneous developmental trajectory of PPC
  - Neural representations of PPC converge between distinct problem types.
- Children with developmental dyscalculia
  - Show persistent deficit in mathematical skill
  - fail to generate distinct representation between different problem types.

Acknowledgement

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Stanford Cognitive & System Neuroscience Lab